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Stress state dependence of in-reactor creep and swelling Part I: Continuum plasticity model

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A B S T R A C T

Irradiation induced swelling of reactor core materials may jeopardize safe and reliable operation of fast reactors due to swelling-induced distortion and interference of core components. The principles of incremental continuum plasticity are used here to develop constitutive equations that can be used to conduct engineering evaluations of these potential problems. The equations are used in Part II to analyze previously unreported in-reactor creep and swelling data obtained ca. 1977–1979 as part of the US breeder reactor program. Results of this stress state experiment showed for the first time that a deviatoric stress can affect volumetric swelling. The constitutive equations developed here predict that, in the presence of significant swelling, deviatoric and volumetric strain rate components each are functions of both deviatoric and hydrostatic components of stress for both linear and non-linear creep.

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1. Introduction

Irradiation induced void swelling of fast reactor core materials may lead to distortion and interference of core components, which can affect coolant flow, core reactivity, the movement of control and safety rods and removal of expended fuel bundles. Engineering evaluations of the severity of these potential problems require multiaxial equations that capture the complex relationships that exist among irradiation creep, swelling and the components of stress. Early experimental results showed that stress can increase volumetric swelling rate [1] and that deviatoric, volume conservative creep may be dependent on the swelling rate [2,3].

Experimental evidence that deviatoric stress may affect swelling first was reported in 1979 at a meeting of the US National Cladding and Duct Materials Development Program [4] where preliminary results of an in-reactor multiaxial creep experiment [5] were presented. Experimental results of this "stress state" experiment were summarized by Garner [6] but details were not reported in the open literature due to reductions in funding and eventual cancellation of the US breeder reactor program. Results of the stress state experiment, which are reported in a companion paper [7], hereafter referred to as Part II, show that in the presence of significant swelling, stress-affected swelling rate may depend on deviatoric stress, even for a pure shear stress state for which there is no hydrostatic stress.

Commenting on results of the stress state experiment, Garner [6] conjectured that the deviatoric component of a stress state, rather than the hydrostatic component, is likely the component that affects swelling and that this is an indication of the strong link between swelling and irradiation creep. The effect of a shear (deviatoric) stress on swelling was considered by Straalsund, Guthrie and Wolfer (SGW) [8] in an early void growth model, leading to a general conclusion "that both stress-accelerated void growth and dislocation loop creep are manifestations of the same mechanism, and must therefore occur simultaneously in the presence of a stress state that contains both deviatoric and hydrostatic components". Their model, however, related stress effect on void growth entirely to the hydrostatic stress component and, therefore, did not include the possibility that deviatoric stress may affect void growth in the absence of a hydrostatic component.

The principles of continuum plasticity, sometimes guided by mechanistic models, previously have been used [9–11] to derive multiaxial constitutive equations for neutron irradiated materials undergoing void swelling. By analogy to the multiaxial stress–strain equations for linear elastic behavior, Wire and Straalsund (WS) [10] developed a multiaxial, linear creep equation that includes stress-affected swelling. However, these equations again related stress-affected swelling to the hydrostatic component of stress, only.

Hall [11] extended the WS equations to include non-linear creep of a material undergoing stress-free and stress-affected void swelling. Using microstructural modeling estimates of equation parameters, obtained from the work of SGW [8], the predicted effects of stress state triaxiality, stress exponent and plastic Poisson's ratio on deviatoric strain rate were explored parametrically. Although

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not emphasized in this early paper, Hall's equations also predict that swelling is a function of deviatoric stress, but only for non-linear creep and a for mixed deviatoric plus hydrostatic stress state.

Hall's original constitutive equations are extended here to include the possibility that, in addition to mechanistic coupling of deviatoric creep and stress-affected swelling, there is a stress-state coupling of the deviatoric and hydrostatic stress invariants. This approach provides the desired expression for volumetric strain rate, which predicts that a deviatoric stress may affect swelling in the absence of a hydrostatic stress, for both linear and non-linear creep. The revised deviatoric and volumetric strain rate equations are explored parametrically in order to illustrate the complex, stress-state dependent correlations that potentially may exist among the creep and swelling components of in-reactor deformation and the deviatoric and hydrostatic invariants of the applied stress.

2. Constitutive equation development

2.1. Strain rate and stress components

Constitutive equations relate principal components of irradiation damage-based strain rate, $\dot{\varepsilon}_i (i = 1, 2, 3)$, to principal stresses, $\sigma_i (i = 1, 2, 3)$. Damage-based strain rates are defined as plastic strain increments per atomic displacement, $\dot{\varepsilon}_i \equiv \Delta\varepsilon_i / \Delta dpa,\ (i = 1, 2, 3)$. Each strain rate component is a sum of two terms, $\dot{\varepsilon}_i = \dot{e}'_i + \dot{S}/3,\ (i = 1, 2, 3)$, where $\dot{e}'_i (i = 1, 2, 3)$ are volume conservative, deviatoric components of strain (creep) rate, $\dot{e}'_1 + \dot{e}'_2 + \dot{e}'_3 = 0$, and $\dot{S} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3$ is volume non-conservative swelling rate. Volumetric swelling rate is a sum of two terms, $\dot{S} = \dot{S}_o + \dot{S}_\sigma$, where $\dot{S}_o$ is the stress-free swelling rate, and $\dot{S}_\sigma$ is the stress-affected swelling rate, the latter of which also will be called "volumetric creep", depending on context. The total stress-dependent strain rate is $\dot{e}_i = \dot{e}'_i + \dot{S}_\sigma/3,\ (i = 1, 2, 3)$, or $\dot{e}_i = \dot{\varepsilon}_i - \dot{S}_o/3,\ (i = 1, 2, 3)$. The mean (hydrostatic) stress, is $\sigma_H = (\sigma_1 + \sigma_2 + \sigma_3)/3$ and the deviatoric components of the principal stress are $\sigma'_i = \sigma_i - \sigma_H,\ (i = 1, 2, 3)$, where $\sigma'_1 + \sigma'_2 + \sigma'_3 = 0$.

Equivalent multiaxial stress states are defined as those having equal mechanical energy dissipation rates:

$$\dot{W} \equiv \sum_{i=1}^{3} \sigma_i \dot{e}_i = \sum_{i=1}^{3} (\sigma'_i + \sigma_H)\left(\dot{e}'_i + \dot{S}_\sigma/3\right) = \sigma_e \dot{\varepsilon}_e, \tag{1}$$

where σ_e is the equivalent stress and $\dot{\varepsilon}_e$ is the equivalent strain rate. The uniaxial stress state is chosen as the reference stress state with the requirement that for a tensile test $(i = 1)$, the proportionality between $\dot{\varepsilon}_e$ and σ_e must reduce to $\dot{\varepsilon}_e/\sigma_e = (\dot{\varepsilon}_1 - \dot{S}_o/3)/\sigma_1 = \dot{e}_1/\sigma_1$. The conventional von Mises equivalent stress, σ_{vM}, and equivalent strain rate, $\dot{\varepsilon}_{vM}$, are defined, respectively, as invariants of the deviatoric stress and strain rate tensors:

$$\sigma_{vM} = \sqrt{\frac{3}{2}\sum_{i=1}^{3} \sigma'_i \sigma'_i}; \quad \dot{\varepsilon}_{vM} = \sqrt{\frac{2}{3}\sum_{i=1}^{3} \dot{e}'_i \dot{e}'}. \tag{2}$$

2.2. Deviatoric and volumetric creep rate equations

Hall's original constitutive strain rate equations were developed according to the principles of incremental, continuum plasticity, beginning with an assumption of the St. Venant–Levy–Mises flow rule, which relates increments of plastic strain to stress [12]. The principal shear strain increments were assumed to be proportional to principal components of shear stress and the stress-affected swelling increment was assumed to be proportional to the hydrostatic stress. Equivalent multiaxial stress states were defined according to Eq. (1) and the uniaxial stress state was chosen as the reference stress state. With these assumptions and requirements, expressions were derived for σ_e and $\dot{\varepsilon}_e$. Deviatoric and volumetric strain rate equations were then derived by substitution back into the St. Venant–Levy–Mises equations.

Deviatoric and volumetric creep rate equations also can be obtained more generally by assuming the existence of a plastic potential, which is a scalar function of stress from which the plastic strain increments can be obtained by differentiation with respect to the stress components [13]. This function serves the same role as the yield function associated with a yield criterion, such as the distortion energy criterion of von Mises. By assuming a multiaxial equivalent stress, σ_e, as the yield function, the plastic strain increment can be obtained according to

$$\dot{e}'_i = \dot{\varepsilon}_e \frac{\partial \sigma_e}{\partial \sigma'_i},\ i = 1, 2, 3; \quad \dot{S}_\sigma = \dot{\varepsilon}_e \frac{\partial \sigma_e}{\partial \sigma_H}. \tag{3}$$

An expression for σ_e and an expression for $\dot{\varepsilon}_e$ in terms of σ_e are needed to apply these equations. We assume that if there is a power law relationship between strain rate and stress for a reference uniaxial stress state, that is, $\dot{e} = \dot{\varepsilon}_o(\sigma/\sigma_o)^n$, then for all stress states $\dot{\varepsilon}_e = \dot{\varepsilon}_o(\sigma_e/\sigma_o)^n$ where $\dot{\varepsilon}_o$ is a reference strain rate and σ_o is a reference stress. In principle, any convenient reference strain rate could be selected. However, as discussed in more detail in Section 2.5, the experimentally observed and theoretically predicted coupling of deviatoric and volumetric creep rates with stress-free swelling rate can be used to establish an expression relating $\dot{\varepsilon}_o$ to the stress-free swelling rate, $\dot{S}_o$, and the swelling-independent creep compliance, B_o.

2.3. Coupling of deviatoric and hydrostatic stress invariants

Derivation of a volumetric creep equation that is a function of the deviatoric stress in the absence of a hydrostatic stress component can be accomplished phenomenologically by extending Hall's effective stress expression [11] to include a term that couples deviatoric and hydrostatic stress invariants:

$$\sigma_e = \left[\frac{2}{3}(1 + v_p)\sigma_{vM}^2 + 3(1 - 2v_p)\sigma_H^2 + 4\lambda(1 + v_p)(1 - 2v_p)\sigma_{vM}\sigma_H\right]^{\frac{1}{2}}. \tag{4}$$

In this expression v_p is a material parameter that, by analogy to linear elasticity, is identified [8,10] as a plastic Poisson's ratio, which is defined as a ratio of the stress-dependent plastic increments of strain in a tensile test, $v_p \equiv -\dot{e}_2/\dot{e}_1$. The third term on the right hand side of Eq. (4) is the stress invariant coupling term and λ is the stress invariant coupling coefficient. When $\lambda = 0$, Eq. (4) reduces to Hall's original effective stress expression, where v_p in this expression is related to a parameter β in the original expression by $v_p = (1 - \beta)/(2 + \beta)$. Regrouping terms, this equation can be rewritten as

$$\sigma_e = \left\{\frac{2}{3}(1 + v_p)\left[1 + 3\lambda(1 - 2v_p)\frac{\sigma_H}{\sigma_{vM}}\right]\sigma_{vM}^2 + 3(1 - 2v_p)\left[1 + \frac{2}{3}\lambda(1 + v_p)\frac{\sigma_{vM}}{\sigma_H}\right]\sigma_H^2\right\}^{\frac{1}{2}}. \tag{5}$$

The factors in square brackets, each of which includes one-half the stress invariant coupling term, are functions of the stress state as expressed by a ratio of stress state invariants called the stress state triaxiality, σ_H/σ_{vM}. The effect of the factors in brackets is to determine the relative contributions of deviatoric and hydrostatic stress components to the equivalent stress. One result of adopting this equation is that the stress invariants are no longer independent stress variables as the contribution of each to σ_e now depends on the magnitude of the other.

Strain rate components are derived from Eq. (5) using Eqs. (3). The deviatoric creep rate is

$$\dot{e}'_i = \dot{\varepsilon}_e \frac{\partial \sigma_e}{\partial \sigma'_i} = (1+\nu_p)\dot{\varepsilon}_o \left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\left[2\lambda(1-2\nu_p)\frac{\sigma_H}{\sigma_o} + \frac{\sigma'_i}{\sigma_o}\right],$$
$$i = 1,2,3, \quad (6)$$

and the volumetric creep rate is

$$\dot{S}_\sigma = \dot{\varepsilon}_e \frac{\partial \sigma_e}{\partial \sigma_H} = 3(1-2\nu_p)\dot{\varepsilon}_o \left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\left[\frac{2}{3}\lambda(1+\nu_p)\frac{\sigma_{vM}}{\sigma_o} + \frac{\sigma_H}{\sigma_o}\right]. \quad (7)$$

When $\lambda = 0$, Eqs. (6) and (7) reduce to Hall's original constitutive equations. In this case, the volumetric creep rate is a function of the deviatoric stress only for non-linear creep ($n > 1$). When $\lambda > 0$, Eq. (6) shows that, even for linear creep ($n = 1$) and as long as $-1 < \nu_p < 1/2$, the deviatoric creep rate is a function of both deviatoric and hydrostatic stress and Eq. (7) shows that the volumetric strain rate is a function of both hydrostatic and deviatoric stress invariants. Moreover Eq. (7) shows the desired result that a deviatoric stress may affect swelling in the absence of a hydrostatic component. For example, for $n = 1$ and $\sigma_H = 0$, Eq. (7) reduces to

$$\dot{S}_\sigma = 2\lambda\dot{\varepsilon}_o(1-2\nu_p)(1+\nu_p)\sigma_{vM}/\sigma_o. \quad (8)$$

Likewise, for $n = 1$ and $\sigma_{vM} = 0$, Eq. (6) reduces to

$$\dot{e}'_i = 2\lambda\dot{\varepsilon}_o(1-2\nu_p)(1+\nu_p)\sigma_H/\sigma_o, \quad i = 1,2,3. \quad (9)$$

Eq. (8) shows that for a pure shear state of stress, for which there is only a deviatoric stress component, there is a volumetric strain rate component as long as $\lambda > 0$ and $-1 < \nu_p < 1/2$. Eq. (9) shows that for a pure hydrostatic state of stress there is a deviatoric creep rate, again, as long as $\lambda > 0$ and $-1 < \nu_p < 1/2$. Finally, note that when $\nu_p = 1/2$, which is the case for creep of void free material, Eq. (8) shows that there is no stress-affected swelling component, as $\dot{S}_\sigma = 0$.

2.4. Relationship to other continuum models

These results are counterintuitive based on the precepts of conventional continuum plasticity. In fact, physical justification for the possibility that deviatoric stress may influence the growth of voids in metals in the absence of a hydrostatic stress may be unique to irradiation induced mechanisms of creep and void swelling. Although models for the densification of porous metals by thermal creep [14] predict that deviatoric stress may affect the rate of densification, they require non-linear, power-law creep and a stress state that is largely hydrostatic. While stress acts directly to shrink pores in metals by a thermally-activated, coupled mechanism of vacancy diffusion and dislocation creep [15], stress acts indirectly to enhance irradiation void growth. Both the deviatoric and hydrostatic components of stress increase the bias for absorption of interstitials by dislocations thereby increasing the concentration of excess vacancies available for absorption by defect-neutral voids, which increases the rate of void growth and couples void growth with volume conservative creep [16]. This is discussed in more detail in Section 2.5.

2.5. Model parameters

There are five parameters, ν_p, n, $\dot{\varepsilon}_o$, σ_o and λ that we expect to have values that are specific to the operative creep mechanisms. Matthews and Finnis [17] summarized the creep mechanisms that have been proposed for irradiation creep. Both swelling-independent and swelling-dependent deviatoric creep mechanisms were considered. According to these authors, stress-induced preferential nucleation (SIPN) of dislocation loops and the bowing of dislocation lines by stress-assisted preferential absorption (SIPA) of interstitials are most relevant to the transient period prior to attainment of steady state creep. Dislocation climb, by stress-induced preferential absorption of interstitials at dislocations followed by dislocation glide are considered to be necessary to account for steady state creep.

In simplest expression these models assume that the only sinks for absorption of interstitials and vacancies are dislocations and voids, that dislocations have a bias for the absorption of interstitials, that voids are neutral sinks for the absorption of interstitials and vacancies, and that creep is by dislocation climb. Both deviatoric and hydrostatic components of stress increase the bias of dislocations for the absorption of interstitials, thereby increasing the absorption of excess vacancies by voids. An applied deviatoric stress, acting alone, may increase dislocation bias for interstitials and thereby indirectly enhance the void growth rate.

With these assumptions, SGW [8] developed approximate expressions for the plastic Poisson's ratio: $\nu_p \equiv -\dot{e}_2/\dot{e}_1 \approx 1/2 - 4\pi r_v\rho_v/\rho_d$ as $4\pi\rho_v r_v/\rho_d \to 0$ and $\nu_p \approx -1/3$ as $4\pi\rho_v r_v/\rho_d \to \infty$. In these expressions, r_v is the average void radius, ρ_v is the number of voids per unit volume and ρ_d is the dislocation line length per unit volume. These expressions predict that $\nu_p = 1/2$ when the void fraction, $4\pi r_v\rho_v$, is zero, which is the expected result for non-irradiated, void free material. With increasing irradiation damage, voids become the dominant sink when $4\pi r_v\rho_v > \rho_d$. As a result, ν_p decreases, and may become negative. According to SGW, there are no combinations of r_v, ρ_v, and ρ_d that will yield a lower value for ν_p than $-1/3$. However, a plastic Poisson's ratio of $\nu_p = -1$ is physically permissible [18] and materials having negative Poisson's ratios, as low as about -0.8, have been manufactured and studied extensively [19]. Negative ratios are of interest here as Eqs. (6) and (7) predict that for $\nu_p = -1$, creep deformation consists of volumetric strain rate only, that is, there is no deviatoric strain rate component. Furthermore, "disappearing creep" has been reported by Porter and Garner [20] for stainless steel irradiated at a temperature of 550 °C to about 60 dpa in EBR-II.

Using experimental measurements of r_v, ρ_v and ρ_d obtained by examination of irradiated solution annealed Type 316 stainless steel, SGW were able to estimate ν_p as a function of irradiation damage. They examined stainless steel that had been irradiated over the temperature range of about 375–650 °C for two neutron fluence levels (3×10^{22} n/cm^2 and 5×10^{22} n/cm^2). They found that for the highest fluence level and a void fraction of 0.02, ν_p had an estimated value of about 0–0.02 over a broad temperature range from about 450 °C to 550 °C. Presumably, further reductions in ν_p are possible with increased swelling levels. Fig. 1 shows the results of the SGW study plotted as ν_p versus irradiation damage in units of dpa. The dashed curve is an extrapolation of the data trend to $\nu_p = -1$, which, to be consistent with the results reported by Porter and Garner, was forced to occur at about 60 dpa.

Not included explicitly in the SGW expressions for ν_p is the effect that the dislocation structure has on the magnitude of ν_p. Their expression for ν_p can be written as $\nu_p \equiv 1/2 - 3Q/2\delta$ where $Q \equiv 4\pi r_v\rho_v Z_v/\rho_d Z_v^d$ is a ratio of void sink strength to dislocation sink strength and $\delta = \Delta Z_i^d/(Z_i^d - Z_v^d)$ is a dislocation alignment bias factor. In this expression ΔZ_i^d is the difference in capture efficiency of stress-aligned and non-aligned dislocations and $Z_{i,v}^d$ are the capture efficiencies of dislocations for interstitials and vacancies. Then ν_p is not simply a function of the ratio of void and dislocation densities but is a function of the relative void and dislocation sink strengths and the dislocation alignment bias factor.

This understanding of ν_p has special significance for reirradiation experiments. For example, under the influence of an anisotropic applied stress state, stress-induced preferential nucleation and alignment of dislocation loops will lead to an anisotropic dislocation structure that can support a deviatoric strain component, even after the stress is removed, as has been shown by Garner, Flinn and Hall [21] in a stress-history effects experiment. Results

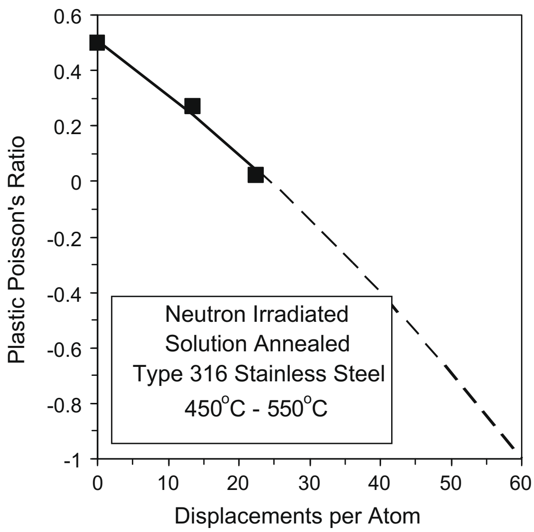

Fig. 1. Plastic Poisson's ratio derived from microstructural data [8]. Data are extrapolated to a value of −1 at 60 dpa in keeping with observations of "disappearing creep" [6].

of this experiment showed that when tubes that were previously irradiated under a 2/1 biaxial stress state were reirradiated without stress the strains accumulated during the reirradiation period were not isotropically distributed, but continued to exhibit strain ratios characteristic of the 2/1 stress state. The anisotropic strain rate diminished with increasing irradiation exposure, however, it is clear that the material retained a memory of the previous stress state.

This creep-before-swelling experiment is the inverse of the experiment reported in Part II, which is a swelling-before-creep experiment. In the present case, with this understanding of the effect of pre-existing dislocation structures, one should expect that application of uniaxial stress state to tubing that has an established isotropic dislocation structure would result in less lateral contraction than would be characteristic of a specimen irradiated with an applied stress from the beginning of the irradiation exposure. This would then appear as a smaller reduction in ν_p than would otherwise occur. Experimental determinations of ν_p using tubing previously irradiated without stress, as in Part II, may then be expected to give values larger than those predicted by the approximate expressions derived by SGW.

Pure climb models of irradiation creep predict that the creep rate is a linear function of stress while for climb-glide models the stress exponent, n, depends on the obstacle to glide [16]. In the case where mobile dislocations are blocked by a pile-up of dislocations against a strong obstacle, n is 2. In the discussion below we assume that n is either 1 or 2.

Wolfer et al. [22] developed a microstructural model from which they derived a semi-empirical expression for deviatoric creep as a simple linear sum of swelling-independent and swelling-dependent terms,

$$\dot{e}_i' = (B_o + D\dot{S})\sigma_i', \quad i = 1, 2, 3, \tag{10}$$

where B_o is the swelling-independent creep compliance and D is the creep-swelling coupling coefficient. This equation is justified phenomenologically as irradiation creep occurs in the absence of swelling as well as after swelling begins [6]. But as $\dot{S} = \dot{S}_o + \dot{S}_\sigma$, and $\dot{S}_\sigma$ arguably is a function of $\dot{S}_o$, Eq. (10) is unnecessarily complicated and can be reduced to a dependence on $\dot{S}_o$. Moreover, this equation is inconsistent with the equations developed here based on considerations of the continuum flow of materials. This can be demonstrated by considering Eqs. (6) and (7), which establish a relationship between $\dot{e}_i'$ and $\dot{S}_\sigma$ that is independent of any relationship there may or may not be between $\dot{e}_i'$ and $\dot{S}$. Combining Eqs. (6) and (7) and solving we find

$$\dot{e}_i' = \frac{1+\nu_p}{3(1-2\nu_p)}\frac{\sigma_i'}{\sigma_H}\dot{S}_\sigma, \quad i = 1, 2, 3, \tag{11}$$

where it is assumed that $n = 1$ and $\lambda = 0$ to achieve comparability to Eq. (10). As Eqs. (10) and (11) cannot hold simultaneously, we reject Eq. (10) as an acceptable expression of the creep-swelling coupling.

In order to maintain an uncomplicated and non-redundant accounting of both swelling-dependent and swelling-independent components of the total irradiation creep deformation, it is advantageous to relate the deviatoric creep components $\dot{e}_i'$ to $\dot{S}_o$ rather than to $\dot{S}$ so that Eq. (10) becomes

$$\dot{e}_i' = (B_o + D\dot{S}_o)\sigma_i', \quad i = 1, 2, 3. \tag{12}$$

This relationship is more accessible experimentally as independent measurements of the stress-free swelling, $\dot{S}_o$, can be obtained using stress-free control specimens.

Note that the constitutive model developed here has to this point not anticipated that there may be two independent types of creep mechanisms, one swelling-independent and the other swelling dependent. In order to incorporate the assumption that these two creep components are independent and additive, and to honor customary usage found in the literature [6], we specialize Eq. (6) for these two cases and then simply add the resulting equations. First consider the conditions for which Eq. (12) was developed, that is, for a uniaxial stress state, $n = 1$, and $\lambda = 0$. In this case Eq. (6) reduces to $\dot{e}_i' = (1+\nu_p)\dot{\varepsilon}_o\sigma_i'/\sigma_o$. For the swelling-independent creep case we note that when $\dot{S}_o = 0$ the irradiation void fraction is negligible and, according to the discussion above, $\nu_p = 1/2$. Then $\dot{e}_i' = 3\dot{\varepsilon}_o\sigma_i'/2\sigma_o$ and $\dot{S}_\sigma = 0$ so that by comparison to Eq. (12), we must have $B_o = 3\dot{\varepsilon}_o/2\sigma_o$. Now for the swelling-dependent creep case we take $\dot{S}_o > 0$, $B_o = 0$ and $-1 < \nu_p < 1/2$. Then $D\dot{S}_o = (1+\nu_p)\dot{\varepsilon}_o/\sigma_o$. If for this case we now choose as the reference creep rate, $\dot{\varepsilon}_o \equiv \dot{S}_o$, we can identify D as $D = (1+\nu_p)/\sigma_o$. Now, consistent with the assumption that swelling-independent and swelling-dependent creep components are additive, the expression for the total deviatoric creep can be written in terms of B_o and D as

$$\dot{e}_i' = B_o\left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\sigma_i' + D\dot{S}_o\left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\left[2\lambda(1-2\nu_p)\sigma_H + \sigma_i'\right],$$
$$i = 1, 2, 3, \tag{13}$$

and the volumetric creep rate, Eq. (7), becomes

$$\dot{S}_\sigma = \frac{3(1-2\nu_p)}{1+\nu_p}D\dot{S}_o\left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\left[\frac{2}{3}\lambda(1+\nu_p)\sigma_{vM} + \sigma_H\right]. \tag{14}$$

Within the assumptions and equations developed here, the magnitude of $D = (1+\nu_p)/\sigma_o$ is expected to decrease with increasing radiation exposure, from an initial value of $D = 1.5/\sigma_o$ to a value of $D = 0.67/\sigma_o$ as ν_p decreases from a maximum value of 1/2 to a value as low as perhaps −1/3. Note that the reference stress, σ_o, which may be considered a radiated-affected flow stress, is expected to increase with increasing radiation exposure, again predicting that D will decrease with increasing radiation exposure. This trend is confirmed by Garner's review and discussion [6] of the collection and interpretation of experimental data from which magnitude of the creep-swelling coupling coefficient D has been derived. In a more recent paper Ukai and Ohtsuka [23] analyzed fuel pin cladding and pressurized tube data obtained on Modified 316 stainless steel and found that D decreased and asymptotically approached a constant value with increasing swelling strain. Using a climb-controlled glide rate-theory model, they also showed that $D \sim 1/\sigma_o$ where $\sigma_o = \alpha b G\sqrt{\rho_d}$, α is a coefficient involving the Taylor factor, b is magnitude of the Burgers vector, G is the shear modulus and ρ_d is the density of dislocations. They attributed the decline in D to the presence of precipitates acting as point defect sinks while here the decline, beyond that due to an increase in σ_o, is attributed to a decrease in ν_p.

There appears to be no direct analogue, in either continuum plasticity or microstructural models of irradiation creep and swelling, to the stress-state dependent terms in Eqs. (6) and (7), or Eqs. (13) and (14), from which the physical meaning and magnitude of the stress invariant coupling parameter λ can be evaluated. The expressions for $\dot{S}_\sigma$ and $\dot{e}'_i$ are however anticipated in the work of Matthews and Finnis [17]. In a manner similar to Eqs. (6) and (7), which are sums of two terms, one proportional to the hydrostatic stress and the other proportional to the deviatoric stress, Matthews and Finnis consider that the bias of dislocations for interstitials can be split into two terms dependent on irradiation damage and stress; one dependent on the hydrostatic stress and one dependent on the deviatoric stress. However, their creep and swelling equations are not easily recast in a manner that reveals a microstructural interpretation of λ. For the parametric studies below, the stress invariant coupling parameter is assumed to have a value $0 \leqslant \lambda \leqslant 1$ where a value of 0 corresponds to no stress invariant coupling.

3. Parametric evaluations

3.1. Deviatoric creep rate

To explore the strain rate behaviors predicted by the constitutive equations developed here, we ignore the uninteresting case of swelling-independent creep (first term on the RHS of Eq. (13)) and focus on the swelling-dependent creep components (Eq. (14) and second term on RHS of Eq. (13)). This is equivalent to exploring Eqs. (6) and (7) with $\dot{\varepsilon}_o \equiv \dot{S}_o$. We also consider the uniaxial and biaxial stress states that can be achieved using tubular specimens subjected to combinations of internal pressure and axial force loading. Table 1 lists stress states, deviatoric stresses, magnitudes of the stress invariants and ratios of these invariants for the stress state experiments reported in Part II. The principal stress components are $\sigma_1 = \sigma$, $\sigma_2 = \alpha\sigma$ and $\sigma_3 = 0$ where σ is the maximum (non-zero) principal stress and α is the ratio of principal stresses that uniquely identifies each biaxial stress state considered. Parametric analyses are performed for each of the stress states, for the corresponding triaxiality ratios of $-1/3$, 0, 1/3 and 2/3, for three values of the coupling parameter, λ = 0, 0.2 and 0.5 and for stress exponents of 1 and 2.

Fig. 2 shows the swelling-dependent deviatoric creep rate, Eq. (6), plotted versus the deviatoric stress for the simplest case; $n = 1$ and $\lambda = 0$. For this case, $\dot{e}'$ is simply proportional to σ' where the proportionality factor decreases as ν_p decreases. Note that this strain rate component "disappears" when $\nu_p = -1$. Fig. 3 shows the effect of stress invariant coupling, that is, for $\lambda > 0$. In this case $\dot{e}'$ is not simply proportional to σ'; it is now a function of the stress state triaxiality, σ_H/σ_{vM}, and is asymmetrical about $\sigma' = 0$, that is, deviatoric strain rate in tension occurs at a greater rate than deviatoric strain rate in compression. This is illustrated for the uniaxial stress state, for which $\sigma_H/\sigma_{vM} = \pm 1/3$. Note also that the deviatoric strain rate component now may "disappear" for uniaxial compression for combinations of λ and ν_p such that $\lambda = 1/(1 - 2\nu_p)$.

Now consider non-linear swelling-dependent deviatoric strain rate. Fig. 4 shows that even in the absence of stress invariant coupling ($\lambda = 0$), $\dot{e}'$ is a function of stress state triaxiality when $n > 1$. However, in this case, strain rate in compression is equal in magnitude to strain rate in tension. Fig. 5 shows that when $n > 1$ and $\lambda > 0$, swelling-dependent deviatoric strain rate is again asymmetric about $\sigma' = 0$.

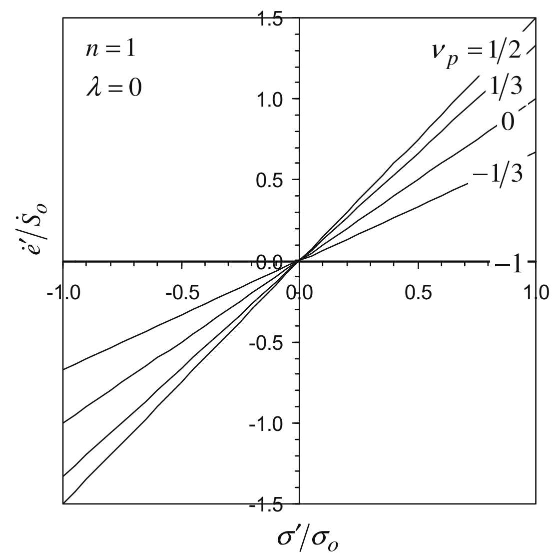

Fig. 2. Swelling-dependent deviatoric creep as a function of deviatoric stress in the absence of stress invariant coupling ($\lambda = 0$) and for a stress exponent of 1. For this case, the swelling-dependent deviatoric creep component is independent of the stress state but has a magnitude that is dependent on the plastic Poisson's Ratio.

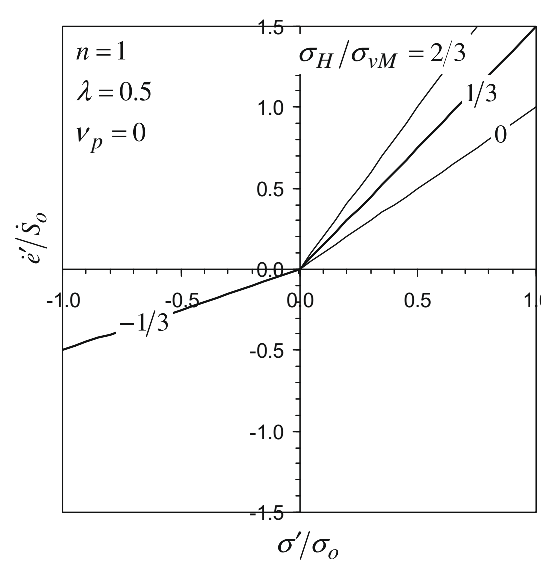

Fig. 3. As in Fig. 2, but with stress invariant coupling ($\lambda = 0.5$) between deviatoric and hydrostatic components of stress. In this case swelling-dependent deviatoric creep is dependent on stress state triaxiality and is asymmetric in deviatoric stress.

Table 1
Stress ratios and stress invariants for representative stress states.

Stress state	$\alpha = \sigma_2/\sigma_1$	σ'	σ_{vM}	σ_H	σ_H/σ_{vM}
Shear	1/−1	σ	$\sqrt{3}\sigma$	0	0
Biaxial tension	2/1	$\sigma/2$	$\sqrt{3}\sigma/2$	$\sigma/2$	$1/\sqrt{3}$
Axial tension	0/1	$2\sigma/3$	σ	$\sigma/3$	1/3
Axial compression	0/−1	σ	σ	$-\sigma/3$	−1/3
Balanced biaxial tension	1/1	$\sigma/3$	σ	$2\sigma/3$	2/3

3.2. Volumetric creep rate

Now consider the effect that deviatoric stress may have on volumetric strain rate, Eq. (7). Fig. 6, which is plotted versus σ_{vM} for $n = 1$ and constant stress state triaxiality ratios, shows that in the absence of stress invariant coupling ($\lambda = 0$), $\dot{S}_\sigma$ is independent of σ_{vM}. That is, $\dot{S}_\sigma$ is simply proportional to σ_H for all stress states and this proportionality increases with stress state triaxiality. The purpose of plotting $\dot{S}_\sigma$ versus σ_{vM} for constant stress state triaxiality ratios instead of plotting versus σ_H is to illustrate under what conditions there may be a deviatoric stress effect in the absence of a hydrostatic stress and to relate this to the effect of other stress states on $\dot{S}_\sigma$. Compare Fig. 6 with Fig. 7, the latter of which is plotted for the same conditions as Fig. 6 except that $\lambda = 0.2$. Fig. 7 shows that when there is a stress invariant coupling ($\lambda > 0$), but there is no hydrostatic stress component, $\dot{S}_\sigma$ is non-zero and increases in proportion to σ_{vM}. As in Fig. 6, the proportionality increases with increasing stress state triaxiality. However, there is now an asymmetry as can be seen by comparing the trends for the uniaxial tension and uniaxial compression stress states. The negative rate of volumetric strain rate for uniaxial compression is less than the positive rate of volumetric strain rate under uniaxial tension. Eq. (7) predicts that the volumetric strain rate component may be suppressed under uniaxial compression for combinations of λ and ν_p such that $1/\lambda = 2(1 + \nu_p)$. As shown in Fig. 8, for larger magnitudes of the stress invariant coupling term ($\lambda = 0.5$), the volumetric strain rate may be positive under uniaxial compression. Fig. 9 shows similar trends for non-linear creep.

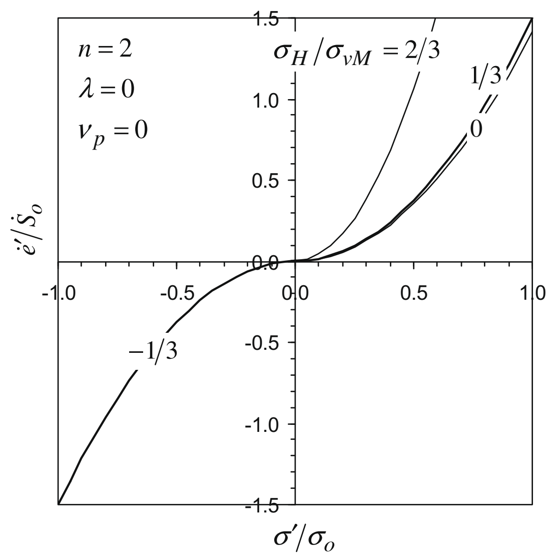

Fig. 4. As in Fig. 2, but for a stress exponent of 2. Swelling-dependent deviatoric creep remains a function of stress state and is symmetric in deviatoric stress.

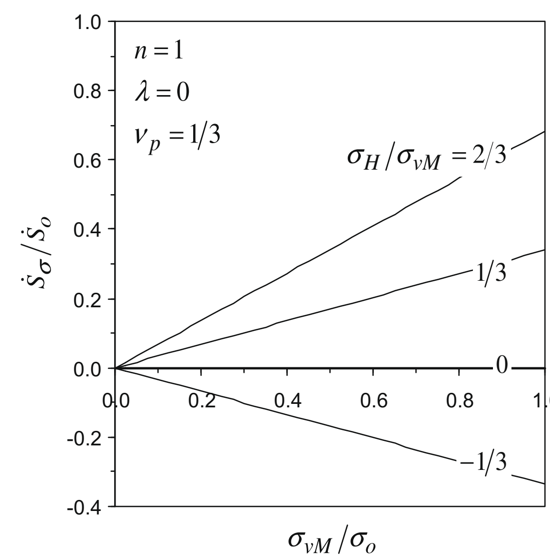

Fig. 6. Volumetric creep as a function of von Mises invariant of the deviatoric stress in the absence of stress invariant coupling ($\lambda = 0$) and for a stress exponent of 1. The curve for a stress state triaxiality of 0 shows that, for this case, volumetric creep is not a function of the deviatoric stress.

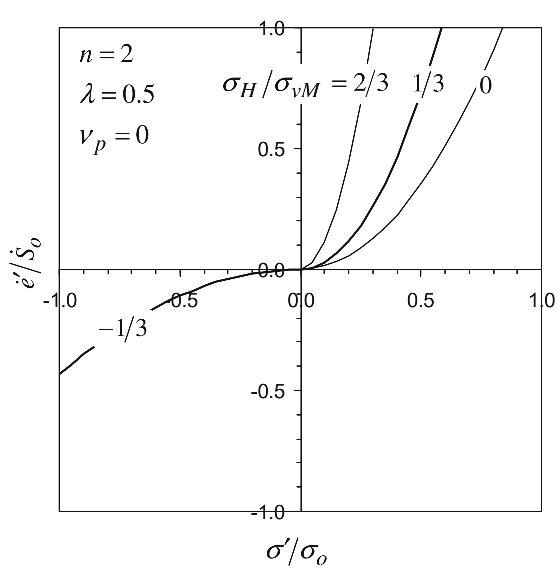

Fig. 5. As in Fig. 4, but with stress invariant coupling ($\lambda = 0.5$). For this case swelling-dependent deviatoric creep is dependent on stress state triaxiality and is asymmetric in deviatoric stress.

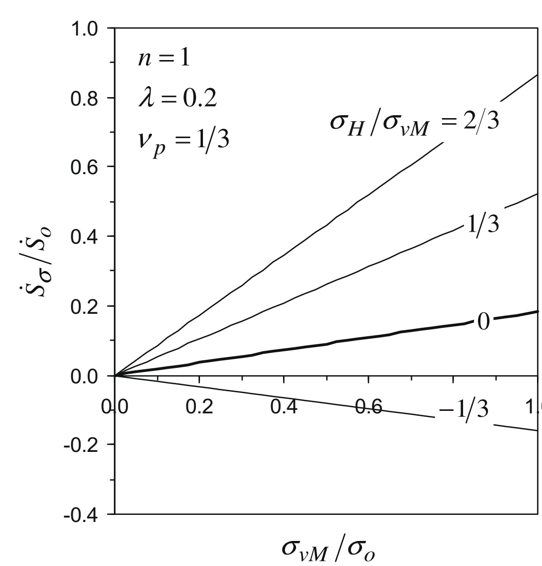

Fig. 7. As in Fig. 6, but for stress invariant coupling ($\lambda = 0.2$). The curve for a stress state triaxiality of 0 shows that, for this case, volumetric creep is a function of the deviatoric stress.

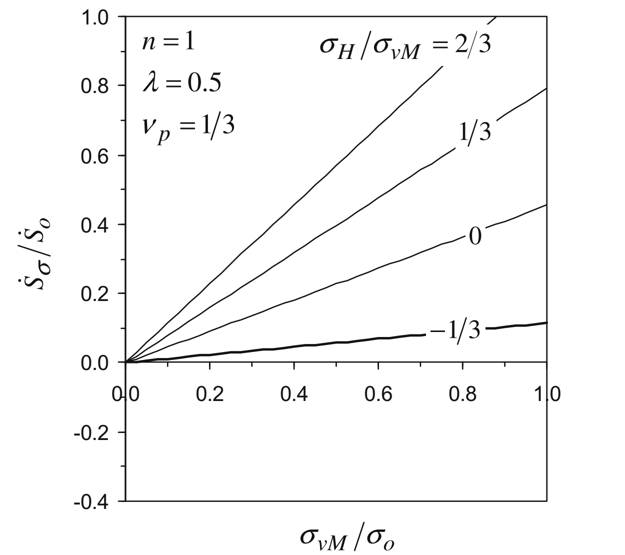

Fig. 8. As in Fig. 7, but for larger stress invariant coupling ($\lambda = 0.5$). The curve for uniaxial compression (triaxiality of $-1/3$) shows that compressive stress states may result in volumetric expansion.

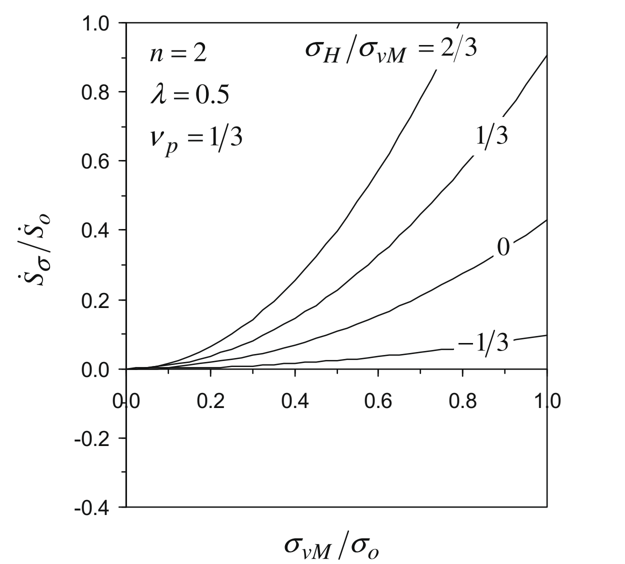

Fig. 9. As in Fig. 8, but for a stress exponent of 2.

4. Summary and conclusions

The constitutive equations developed here provide a phenomenological model that is consistent with the early experimental observation of Hall [11] that a deviatoric stress can affect volumetric creep (stress-affected swelling) in the absence of a hydrostatic stress. The equations moreover predict that both deviatoric and volumetric components of irradiation creep each are functions of both deviatoric and hydrostatic invariants of stress for both linear and non-linear creep.

These constitutive deformation equations appear to have no analogue in conventional continuum plasticity and may be unique to a radiation environment due to the manner with which irradiation creep and swelling are coupled mechanistically. The allocation of irradiation creep between deviatoric and volumetric components is determined by a plastic Poisson's ratio that depends on the relative defect absorption strengths of irradiation voids and dislocations and on isotropy of the dislocation structure. Another unique feature of these equations is the coupling of deviatoric and hydrostatic stress invariants, the strength of which depends on a coupling coefficient that does not have an obvious mechanistic interpretation. Introduction of this stress-state coupling results in the potential for stress-affected swelling for a pure shear, deviatoric stress state and deviatoric creep for a pure hydrostatic stress state. This latter phenomenon would, however, logically require a pre-existing anisotropic dislocation structure. The possibility that this ratio may be negative allows for the possibility that the magnitude of the deviatoric creep rate may be reduced to near zero ("disappear") as the density of irradiation voids approaches and exceeds the dislocation line density. Finally, due to the stress invariant coupling, both deviatoric and volumetric creep occur at rates in tension that are greater than the magnitude of creep rates in compression.

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